THE COSMIC COINCIDENCE AS A TEMPORAL SELECTION EFFECT PRODUCED BY THE AGE DISTRIBUTION OF TERRESTRIAL PLANETS IN THE UNIVERSE

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ABSTRACT

The energy densities of matter and the vacuum are currently observed to be of the same order of magnitude: $(\Omega_{m_0} \approx 0.3) \sim (\Omega_{\Lambda_0} \approx 0.7)$. The cosmological window of time during which this occurs is relatively narrow. Thus, we are presented with the cosmological coincidence problem: why, just now, do these energy densities happen to be of the same order? Here we show that this apparent coincidence can be explained as a temporal selection effect produced by the age distribution of terrestrial planets in the universe. We find a large ($\sim 68\%$) probability that observations made from terrestrial planets will result in finding Ω_m at least as close to Ω_Λ as we observe today. Hence, we, and any observers in the universe who have evolved on terrestrial planets, should not be surprised to find $\Omega_{m_0} \sim \Omega_{\Lambda_0}$. This result is relatively robust if the time it takes an observer to evolve on a terrestrial planet is less than ~ 10 Gyr.

Subject headings: astrobiology — cosmological parameters — cosmology: theory — extraterrestrial intelligence — planetary systems: formation — stars: formation

1. IS THE COSMIC COINCIDENCE REMARKABLE OR INSIGNIFICANT?

1.1. *Dicke's Argument*

Dirac (1937) pointed out the near-equality of several large fundamental dimensionless numbers of the order 10^{40}. One of these large numbers varied with time, since it depended on the age of the universe. Thus, there was a limited time during which this near-equality would hold. Under the assumption that observers could exist at any time during the history of the universe, this large number coincidence could not be explained in the standard cosmology. This problem motivated Dirac (1938) and Jordan (1955) to construct an ad hoc new cosmology. Alternatively, Dicke (1961) proposed that our observations of the universe could only be made during a time interval after carbon had been produced in the universe and before the last stars stop shining. Dicke concluded that this temporal observational selection effect — even one so loosely delimited — could explain Dirac's large number coincidence without invoking a new cosmology.

Here we construct a similar argument to address the cosmic coincidence: why just now do we find ourselves in the relatively brief interval during which $\Omega_m \sim \Omega_\Lambda$? The temporal constraints on observers that we present are more empirical and specific than those used in Dicke's analysis, but the reasoning is similar. Our conclusion is also similar: a temporal observational selection effect can explain the apparent cosmic coincidence. That is, given the evolution of Ω_Λ and Ω_m in our universe, most observers in our universe who have emerged on terrestrial planets will find $\Omega_\Lambda \sim \Omega_m$. Rather than being an unusual coincidence, it is what one should expect.

There are two distinct problems associated with the cosmological constant (Weinberg 2001; Garriga & Vilenkin 2001; Steinhardt 2003). One is the smallness problem and has to do with the observed energy density of the vacuum, ρ_Λ. Why is ρ_Λ so small compared to the $\sim 10^{120}$ times larger value predicted by particle physics? Anthropic solutions to this problem invoke a multiverse and argue that galaxies would not form and there would be no life in a universe, if ρ_Λ were larger than ~ 100 times its observed value (Weinberg 1987; Martel et al. 1998; Garriga & Vilenkin 2001; Pogosian & Vilenkin 2007). Such explanations for the smallness of ρ_Λ do not explain the temporal coincidence between the time of our observation and the time of the near-equality of Ω_m and Ω_Λ. One can address this coincidence problem in the context of a multiverse scenario in which the value of ρ_Λ is treated as a variable while holding other parameters fixed (Garriga et al. 2000). This changes the time t_Λ when ρ_Λ begins to dominate the energy density of the universe. Here we address the coincidence problem in a more restricted context. We consider only the observed universe. Specifically, given the value of ρ_Λ in our universe, why is it that we are here now to observe the start of its dominance?

1.2. *Evolution of the Energy Densities*

Given the currently observed values for H_0 and the energy densities Ω_{r_0}, Ω_{m_0}, and Ω_{Λ_0} in the universe (Spergel et al. 2007; Seljak et al. 2006), the Friedmann equation tells us the evolution of the scale factor a, and the evolution of these energy densities. These are plotted in Figure 1. The history of the universe can be divided chronologically into four distinct periods, each dominated by a different form of energy: initially the false vacuum energy of inflation dominates, then radiation, then matter, and finally vacuum energy (see Table 1). Currently, the universe is making the transition from matter domination to vacuum energy domination. In an expanding universe, with an initial condition $\Omega_m > \Omega_\Lambda > 0$, there will be some epoch in which $\Omega_m \sim \Omega_\Lambda$, since ρ_m is decreasing as $\propto 1/a^3$, while ρ_Λ is a constant (see Fig. 1, *top*, and the Appendix). Figure 1 also shows that the transition from matter domination to vacuum energy domination is occurring now. When we view this transition in the context of the time evolution of the universe (Fig. 2) we are presented with the cosmic coincidence problem: why just now do we find ourselves at the relatively brief interval during which this transition happens? Carroll (2001a, 2001b) and Dodelson et al. (2000) find this coincidence to be a remarkable result that it is crucial to understand.

The cosmic coincidence problem is often regarded as an important unsolved problem whose solution may help unravel the

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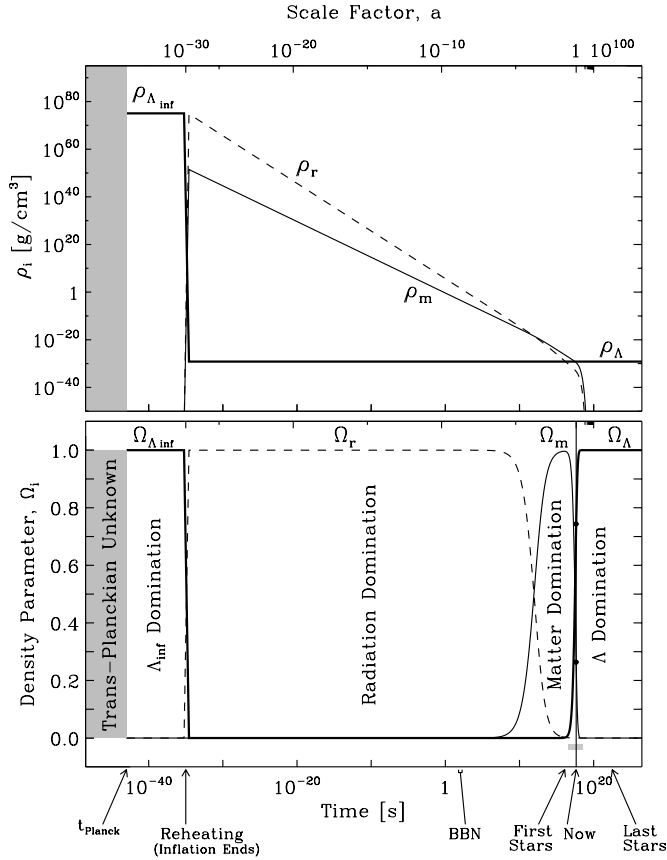[2] Department of Astrophysics, School of Physics, University of New South Wales, Sydney, NSW 2052, Australia.

FIG. 1.—Time dependence of the densities of the major components of the universe. Given the observed Hubble constant, H_0, and energy densities in the universe today, Ω_{r_0}, Ω_{m_0}, and Ω_{Λ_0} (radiation, matter, and cosmological constant), we use the Friedmann equation to plot the temporal evolution of the components of the universe in g cm^{-3} (top), or normalized to the time-dependent critical density $\rho_{crit} = 3H(t)^2/8\pi G$ (bottom). We assume an epoch of inflation at $\sim 10^{-35}$ s after the big bang and a false vacuum energy density $\rho_{\Lambda_{inf}}$ between the Planck scale and t_{GUT}. See Table 1 and the Appendix for details.

nature of dark energy (Turner 2001; Carroll 2001a). The coincidence problem is one of the main motivations for the tracker potentials of quintessence models (Caldwell et al. 1998; Steinhardt et al. 1999; Zlatev et al. 1999; Wang et al. 2000; Dodelson et al. 2000; Armendariz-Picon et al. 2000; Guo & Zhang 2005). In these models the cosmological constant is replaced by a more generic form of dark energy in which Ω_m and Ω_Λ are in near-equality for extended periods of time. It is not clear that these models successfully explain the coincidence without fine-tuning (see Weinberg 2000; Bludman 2004).

The interpretation of the observation $\Omega_{m_0} \sim \Omega_{\Lambda_0}$ as a remarkable coincidence in need of explanation depends on some assumptions that we quantify to determine how surprising this apparent coincidence is. We begin this quantification by introducing a time-dependent proximity parameter,

$$r = \min\left[\frac{\Omega_\Lambda}{\Omega_m}, \frac{\Omega_m}{\Omega_\Lambda}\right], \qquad (1)$$

which is equal to 1 when $\Omega_m = \Omega_\Lambda$ and is close to zero when $\Omega_m \gg \Omega_\Lambda$ or $\Omega_m \ll \Omega_\Lambda$. The current value is $r_0 \approx 0.4$. In Figure 2 we plot r as a function of log (scale factor) in the top panel and as a function of log (time) in the bottom panel. These logarithmic axes allow a large dynamic range that makes our existence at

a time when $r \sim 1$ appear to be an unlikely coincidence. This appearance depends on the implicit assumption that we could make cosmological observations at any time with equal likelihood. More specifically, the implicit assumption is that the a priori probability distribution P_{obs} of the times we could have made our observations is uniform in log t or log a over the interval shown.

Our ability to quantify the significance of the coincidence depends on whether we assume that P_{obs} is uniform in time, log (time), scale factor, or log (scale factor). That is, our result depends on whether we assume $P_{obs}(t) = $ constant, $P_{obs}(\log t) = $ constant, $P_{obs}(a) = $ constant, or $P_{obs}(\log a) = $ constant. These are the most common possibilities, but there are others. For a discussion of the relative merits of log and linear timescales and implicit uniform priors, see § 3.3 and Jaynes (1968).

In Figure 3 we plot $r(t)$ on an axis linear in time where the implicit assumption is that the a priori probability distribution of our existence is uniform in t over the intervals $[0, 100]$ Gyr (top panel) and $[0, 13.8]$ Gyr (bottom panel). The bottom panel shows that the observation $r > 0.4$ could have been made anytime during the past 7.8 Gyr. Thus, our current observation that $r_0 \approx 0.4$ does not appear to be a remarkable coincidence. Whether this most recent 7.8 Gyr period is seen as "brief" (in which case there is an unlikely coincidence in need of explanation) or "long" (in which case there is no coincidence to explain) depends on whether we view the issue in log time (Fig. 2) or linear time (Fig. 3).

A large dynamic range is necessary to present the fundamental changes that occurred in the very early universe, e.g., the transitions at the Planck time, inflation, baryogenesis, nucleosynthesis, recombination, and the formation of the first stars. Thus, a logarithmic time axis is often preferred by early universe cosmologists because it seems obvious, from the point of view of fundamental physics, that the cosmological clock ticks logarithmically. This defensible view and the associated logarithmic axis gives the impression that there is a coincidence in need of an explanation. The linear time axis gives a somewhat different impression. Evidently, deciding whether a coincidence is of some significance or only an accident is not easy (Peebles 1999). We conclude that although the importance of the cosmic coincidence problem is subjective, it is important enough to merit the analysis we perform here.

The interpretation of the observation $\Omega_{m_0} \sim \Omega_{\Lambda_0}$ as a coincidence in need of explanation depends on the a priori (not necessarily uniform) probability distribution of our existence. That is, it depends on when cosmological observers can exist. We propose that the cosmic coincidence problem can be more constructively evaluated by replacing these uninformed uniform priors with the more realistic assumption that observers capable of measuring cosmological parameters are dependent on the emergence of high-density regions of the universe called terrestrial planets, which require nontrivial amounts of time to form—and that once these planets are in place, the observers themselves require nontrivial amounts of time to evolve.

In this paper we use the age distribution of terrestrial planets estimated by Lineweaver (2001) to constrain when in the history of the universe observers on terrestrial planets can exist. In § 2 we briefly describe this age distribution (Fig. 4) and show how it limits the existence of such observers to an interval in which $\Omega_m \sim \Omega_\Lambda$ (Fig. 5). Using this age distribution as a temporal selection function, we compute the probability of an observer on a terrestrial planet observing $r \geq r_0$ (Fig. 6). In § 3 we discuss the robustness of our result and find (Fig. 7) that this result is relatively robust if the time it takes an observer to evolve on a terrestrial planet is less than ~ 10 Gyr. In § 4 we discuss and summarize our results, and compare them to previous work to resolve the

TABLE 1
Important Times in the History of the Universe (Some Used in Table 2)

Event (ref)	Symbol	Time after Big Bang	
		(seconds)	(Gyr)
Planck time, beginning of time	t_{Planck}	5.4×10^{-44}	1.7×10^{-60}
End of inflation, reheating, origin of matter, thermalization	t_{reheat}	$[10^{-43}, 10^{-33}]$	$[10^{-60}, 10^{-50}]$
Energy scale of Grand Unification Theories (GUT)	t_{GUT}	10^{-33}	10^{-50}
Matter-antimatter annihilation, baryogenesis	$t_{baryogenesis}$	$[10^{-33}, 10^{-12}]$	$[10^{-50}, 10^{-29}]$
Electromagnetic and weak nuclear forces diverge	$t_{electroweak}$	10^{-12}	10^{-29}
Light atomic nuclei produced	t_{BBN}	$[100, 300]$	$[3, 9] \times 10^{-15}$
Radiation-matter equality[a]	t_{r-m}	8.9×10^{11}	2.8×10^{-5}
Recombination[a] (first chemistry)	t_{rec}	1.2×10^{13}	0.38×10^{-3}
First thermal disequilibrium	$t_{1st\ therm-dis}$	1.2×10^{13}	0.38×10^{-3}
First stars, Pop III, reionization[a]	$t_{1st\ stars}$	1×10^{16}	0.4
First terrestrial planets[b]	$t_{1st\ Earths}$	8×10^{16}	2.5
Last time r had same value as today	$t_{r\ now}$	1.9×10^{17}	6.1
Formation of the Sun, Earth[c]	t_{Sun}, t_{Earth}	2.9×10^{17}	9.1
Matter-Λ equality[d]	$t_{m-\Lambda}$	3.0×10^{17}	9.4
Now	t_0	4.4×10^{17}	13.8
Last stars die[d]	$t_{last\ stars}$	10^{22}	10^6
Protons decay[d]	$t_{proton\ decay}$	10^{45}	10^{29}
Supermassive black holes consume matter[d]	$t_{black\ holes}$	10^{107}	10^{91}
Maximum entropy (no gradients to drive life)[d]	$t_{heat\ death}$	10^{207}	10^{191}

[a] Spergel et al. (2007); http://map.gsfc.nasa.gov.
[b] Lineweaver (2001).
[c] Allègre et al. (1995).
[d] Adams & Laughlin (1997).

cosmic coincidence problem (Garriga et al. 2000; Bludman & Roos 2001).

2. HOW WE COMPUTE THE PROBABILITY OF OBSERVING $\Omega_m \sim \Omega_\Lambda$

2.1. *The Age Distribution of Terrestrial Planets and New Observers*

The mass histogram of detected extrasolar planets peaks at low masses: $dN/dM \propto M^{-1.7}$, suggesting that low-mass planets are abundant (Lineweaver & Grether 2003). Terrestrial planet formation may be a common feature of star formation (Wetherill 1996; Chyba 1999; Ida & Lin 2005). Whether terrestrial planets are common or rare, they will have an age distribution proportional to the star formation rate (SFR)—modified by the fact that in the first ~2 billion years of star formation, metallicities are so low that the material for terrestrial planet formation will not be readily available. Using these considerations, Lineweaver (2001) estimated the age distribution of terrestrial planets—how many Earths are produced by the universe per year, per Mpc^3 (Fig. 4).

The shape of this distribution is largely determined by the decreasing SFR, since at least a redshift of ~2 (for the past ~8 Gyr). The shape of the distribution does not depend on the somewhat controversial suppression of terrestrial planet formation by hot Jupiters because this effect (if it exists; Mandell et al. 2007) only affects a small number of the highest metallicity systems. If life emerges rapidly on terrestrial planets (Lineweaver & Davis 2002), then this age distribution is the age distribution of biogenesis in the universe.

However, we are not just interested in any life; we would like to know the distribution in time of when independent observers first emerge and are able to measure Ω_m and Ω_Λ, as we are able to do now. If life originates and evolves preferentially on terrestrial planets, then the Lineweaver (2001) estimate of the age distribu-

tion of terrestrial planets is an a priori input which can guide our expectations of when we (as members of a hypothetical group of terrestrial-planet-bound observers) could have been present in the universe. It takes time (if it happens at all) for life to emerge on a new terrestrial planet and evolve into cosmologists who can observe Ω_m and Ω_Λ. Therefore, to obtain the age distribution of new independent observers able to measure the composition of the universe for the first time, we need to shift the age distribution of terrestrial planets by some characteristic time Δt_{obs} required for observers to evolve. On Earth, it took $\Delta t_{obs} \sim 4$ Gyr for this to happen. Whether this is characteristic of life elsewhere in the universe is uncertain (Carter 1983; Lineweaver & Davis 2003). For our initial analysis we use $\Delta t_{obs} = 4$ Gyr as a nominal time to evolve observers. In § 3.1 we allow Δt_{obs} to vary from 0 to 12 Gyr to see how sensitive our result is to these variations. Figure 4 shows the age distribution of terrestrial planet formation in the universe shifted by $\Delta t_{obs} = 4$ Gyr. This curve, labeled "P_{obs}" is a crude prior for the temporal selection effect of when independent observers can first measure r. Thus, if the evolution of biological equipment capable of doing cosmology takes about $\Delta t_{obs} \sim 4$ Gyr, the "P_{obs}" in Figure 4 shows the age distribution of the first cosmologists on terrestrial planets able to look at the universe and determine the overall energy budget, just as we have recently been able to do.

2.2. *The Probability of Observing $\Omega_m \sim \Omega_\Lambda$*

In Figure 5 we zoom into the portion of Figure 1 containing the relatively narrow window of time in which $\Omega_m \sim \Omega_\Lambda$. We plot $r(t)$ to show where $r \sim 1$, and we also plot the age distribution of planets and the age distribution of recently emerged cosmologists from Figure 4. The white area under the thick $P_{obs}(t)$ curve provides an estimate of the time distribution of new observers in the universe. We interpret $P_{obs}(t)$ as the probability distribution of the times at which new, independent observers are able to measure r for the first time.
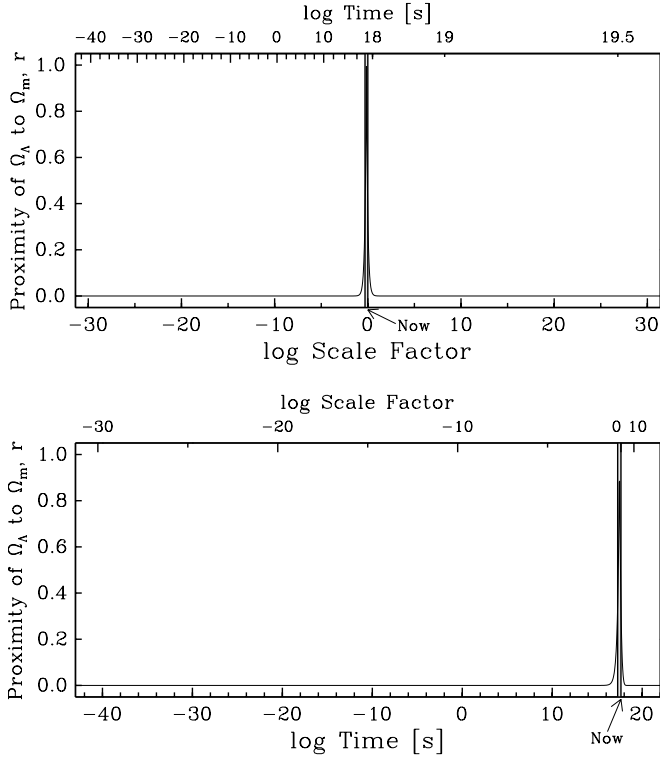
FIG. 2.—Plot of the proximity factor r (see eq. [1]). When the matter and vacuum energy densities of the universe are the same ($\Omega_m = \Omega_\Lambda$), we have $r = 1$. We currently observe $\Omega_{m_0} \sim \Omega_{\Lambda_0}$ and, thus, $r \sim 1$. Our existence now when $r \sim 1$ appears to be an unlikely cosmic coincidence when the x-axis is logarithmic in the scale factor (*top*) or logarithmic in time (*bottom*). In the top panel, following Carroll (2001b), we have chosen a range of scale factors with "Now" midway between the scale factor at the Planck time and the scale factor at the inverse Planck time ($a_{\rm Planck} < a < a_{\rm Planck}^{-1}$). The brief epoch shown in gray between the thin vertical lines is the epoch during which $r > r_0$ (where $r_0 \approx 0.4$ is the currently observed value). In the bottom panel the range shown on the x-axis is $t_{\rm Planck} < t < 10^{22}$ s. The Planck time and Planck scale provide reasonably objective lower time limits. The upper limits are somewhat arbitrary but contribute to the impression that $r \approx 0.4 \sim 1$ is an unlikely coincidence.
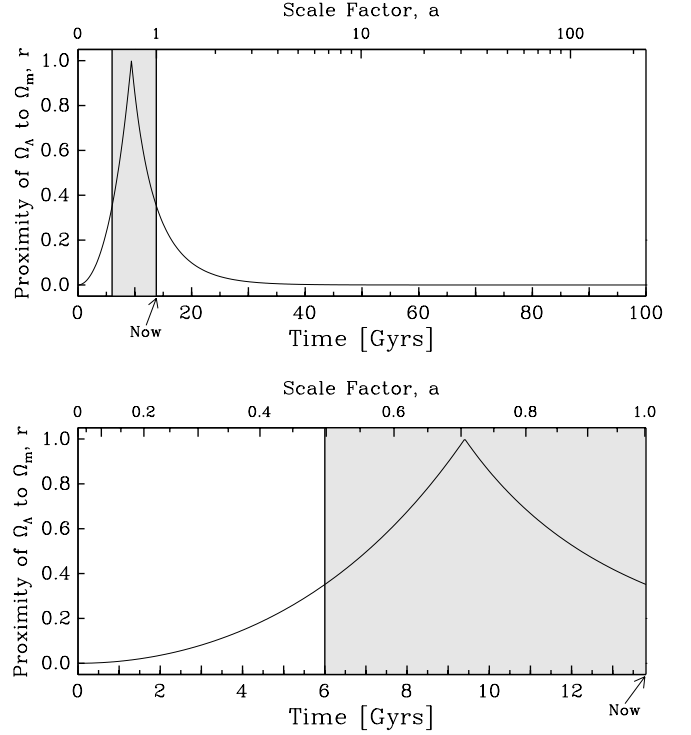

FIG. 3.—Plot of the proximity factor r, as in the previous figure, but plotted here with a linear rather than a logarithmic time axis. The condition $r > r_0 \approx 0.4$ does not seem as unlikely as in the previous figure. The range of time plotted also affects this appearance; with the [0, 100] Gyr range of the top panel, the time interval highlighted in gray (where $r > r_0$) appears narrow and relatively unlikely. In contrast, the [0, 13.8] Gyr range of the bottom panel seems to remove the appearance of $r > r_0$ being an unlikely coincidence; for the first $\sim$6 Gyr we have $r < r_0$, while in the subsequent 7.8 Gyr we have $r > r_0$. How can $r > r_0$ be an unlikely coincidence when it has been true for most of the history of the universe?
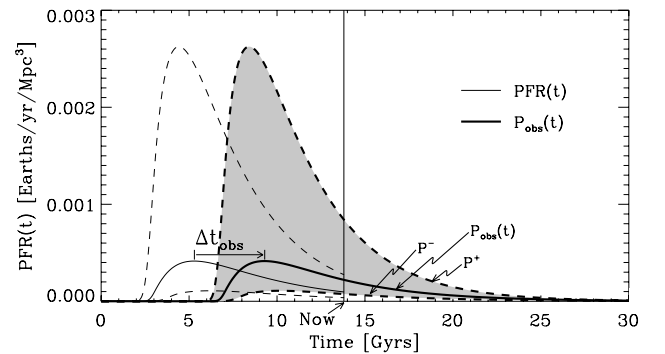
Lineweaver (2001) estimated that the Earth is relatively young compared to other terrestrial planets in the universe. It follows under the simple assumptions of our analysis that most terrestrial-planet-bound observers will emerge earlier than we have. We compute the fraction f of observers who have emerged earlier than we have,

$$f = \frac{\int_0^{t_0} P_{\rm obs}(t)\,dt}{\int_0^{\infty} P_{\rm obs}(t)\,dt} \approx 68\%, \qquad (2)$$

and find that 68% emerge earlier, while 32% emerge later. These numbers are indicated in Figure 5.

2.3. Converting $P_{\rm obs}(t)$ to $P_{\rm obs}(r)$

We have an estimate of the distribution in time of observers, $P_{\rm obs}(t)$, and we have the proximity parameter $r(t)$. We can then convert these to a probability $P_{\rm obs}(r)$ of observed values of r. That is, we change variables and convert the t-dependent probability to an r-dependent probability: $P_{\rm obs}(t) \rightarrow P_{\rm obs}(r)$. We want the probability distribution of the r values first observed by new observers in the universe. Let the probability of observing r in the interval dr be $P_{\rm obs}(r)\,dr$. This is equal to the probability of observing t in the interval dt, which is $P_{\rm obs}(t)\,dt$.

FIG. 4.—Terrestrial planet formation rate PFR(t) (*thin solid line*), derived in Lineweaver (2001). This is an estimate of the age distribution of terrestrial planets in the universe. The estimated uncertainty, based on the uncertainty of the SFR, is given by the thin dashed lines. To allow time for the evolution of observers on terrestrial planets, we shift this distribution by $\Delta t_{\rm obs}$ to obtain an estimate of the age distribution of observers: $P_{\rm obs}(t) = {\rm PFR}(t - \Delta t_{\rm obs})$ (*thick solid line*). The gray band represents the error estimate on $P_{\rm obs}(t)$, which is the shifted error estimate on PFR(t). In the case shown here $\Delta t_{\rm obs} = 4$ Gyr, which is how long it took life on Earth to emerge, evolve, and be able to measure the composition of the universe. To obtain the numerical values on the y-axis, we have followed Lineweaver (2001) and assumed that 1 out of 100 stars is orbited by a terrestrial planet. We have smoothly extrapolated the PFR(t) of Lineweaver (2001) into the future. This time dependence and our subsequent analysis do not depend on whether the probability for terrestrial planets to produce observers is high or low.
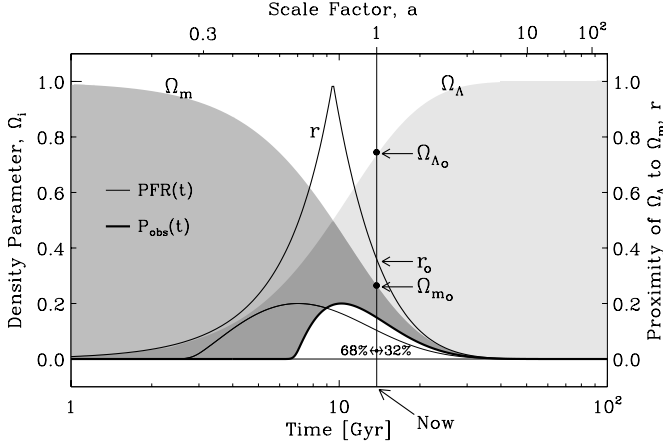
FIG. 5.—Zoom-in of the portion of Fig. 1 between 1 and 100 billion years after the big bang, containing the relatively narrow window of time in which $\Omega_m \sim \Omega_\Lambda$. The 99 Gyr time interval displayed here is indicated in Fig. 1 by the small gray rectangle above the "Now" label. The proximity parameter $r(t)$ (eq. [1], Figs. 2 and 3) is superimposed. The thin solid curve shows the age distribution of terrestrial planets in the universe, while the thick solid curve is the lateral displacement of this distribution by $\Delta t_{obs} = 4$ Gyr. These distributions were presented in Fig. 4, but here the time axis is logarithmic. We interpret P_{obs} as the frequency distribution of new observers able to measure Ω_m and Ω_Λ for the first time. Since $r(t)$ peaks at about the same time as $P_{obs}(t)$, large values of r will be observed more often than small values.

Thus,

$$P_{obs}(r)\,dr = P_{obs}(t)\,dt, \qquad (3)$$

or, equivalently,

$$P_{obs}(r) = \frac{P_{obs}(t)}{dr/dt}, \qquad (4)$$

where $P_{obs}(t) = \mathrm{PFR}(t - \Delta t_{obs})$ is the temporally shifted age distribution of terrestrial planets and dr/dt is the slope of $r(t)$. Both are shown in Figure 5. The distribution $P_{obs}(r)$ is shown in
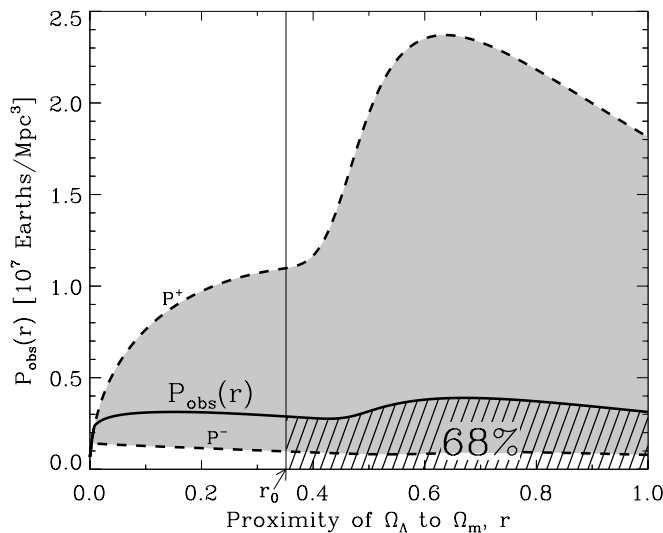

FIG. 6.—Probability of new observers on terrestrial planets observing a given r (eq. [4]). Given our estimate of the age distribution of new cosmologists in the universe $P_{obs}(t)$, the probability of observing Ω_m and Ω_Λ as close together as they are, or closer, is the integral given in eq. (5), shown here as the hashed area labeled 68%. The dashed lines labeled P^+ and P^- arise from replacing $P_{obs}(t)$ in eq. (4) with the curves labeled P^+ and P^- in Fig. 4.
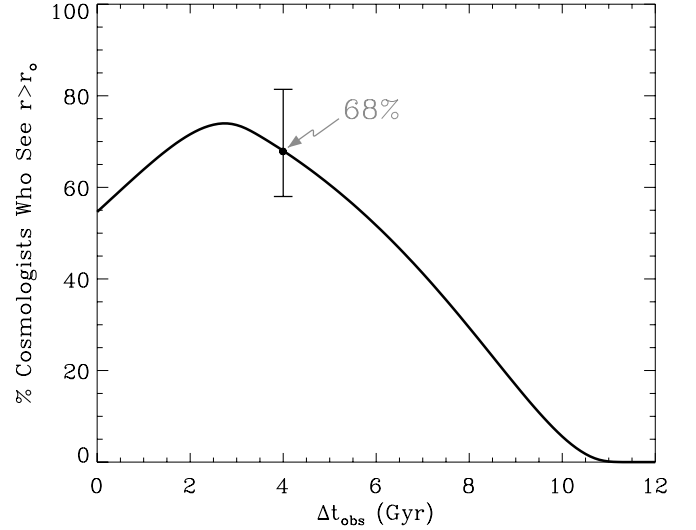

FIG. 7.—Percentage of cosmologists who see $r > r_0$ as a function of the time Δt_{obs} it takes observers to evolve on a terrestrial planet. Since we have only vague notions about how long it takes observers to evolve on a planet, we vary Δt_{obs} between 0 and 12 billion years and show how the probability $P(r > r_0)$ of observing $r > r_0$ (eq. [5]) varies as a function of Δt_{obs}. The $68\%^{+14\%}_{-10\%}$ point plotted is the result from Fig. 6 where $\Delta t_{obs} = 4$ Gyr. If $\Delta t_{obs} = 0$, we use the thin solid curve in Fig. 5 as $P_{obs}(t)$ rather than the thick solid curve, and we obtain 55%.

Figure 6, along with the upper and lower confidence limits on $P_{obs}(r)$ obtained by inserting the upper and lower confidence limits of $P_{obs}(t)$ (denoted "P^+" and "P^-" in Fig. 4) into equation (4) in place of $P_{obs}(t)$.

The probability of observing $r > r_0$ is

$$P(r > r_0) = \int_{r_0}^{1} P_{obs}(r)\,dr = \int_{t'}^{t_0} P_{obs}(t)\,dt \approx 68\%, \quad (5)$$

where t' is the time in the past when r was equal to its present value, i.e., $r(t') = r(t_0) = r_0 \approx 0.4$. We have $t' = 6$ Gyr and $t_0 = 13.8$ Gyr (see Fig. 3, *bottom*). This integral is shown graphically in Figure 6 as the hatched area underneath the "$P_{obs}(r)$" curve, between $r = r_0$ and $r = 1$. We interpret this as follows: of all observers that have emerged on terrestrial planets, 68% will emerge when $r > r_0$ and thus will find $r > r_0$. The 68% from equation (2) is only the same as the 68% from equation (5) because all observers who emerge earlier than we did, did so more recently than 7.8 billion years ago and, thus, observe $r > r_0$ (Fig. 5).

We obtain estimates of the uncertainty on this 68% estimate by computing analogous integrals underneath the curves labeled P^+ and P^- in Figure 6. These yield 82% and 59%, respectively. Thus, under the assumptions made here, $68\%^{+14\%}_{-10\%}$ of the observers in the universe will find Ω_Λ and Ω_m even closer to each other than we do. This suggests that a temporal selection effect due to the constraints on the emergence of observers on terrestrial planets provides a plausible solution to the cosmic coincidence problem. If observers in our universe evolve predominantly on Earth-like planets (see the "principle of mediocrity" in Vilenkin 1995b), we should not be surprised to find ourselves on an Earth-like planet and we should not be surprised to find $\Omega_{\Lambda_0} \sim \Omega_{m_0}$.

3. HOW ROBUST IS THIS 68% RESULT?

3.1. *Dependence on the Timescale for the Evolution of Observers*

A necessary delay, required for the biological evolution of observing equipment—e.g., brains, eyes, and telescopes—makes
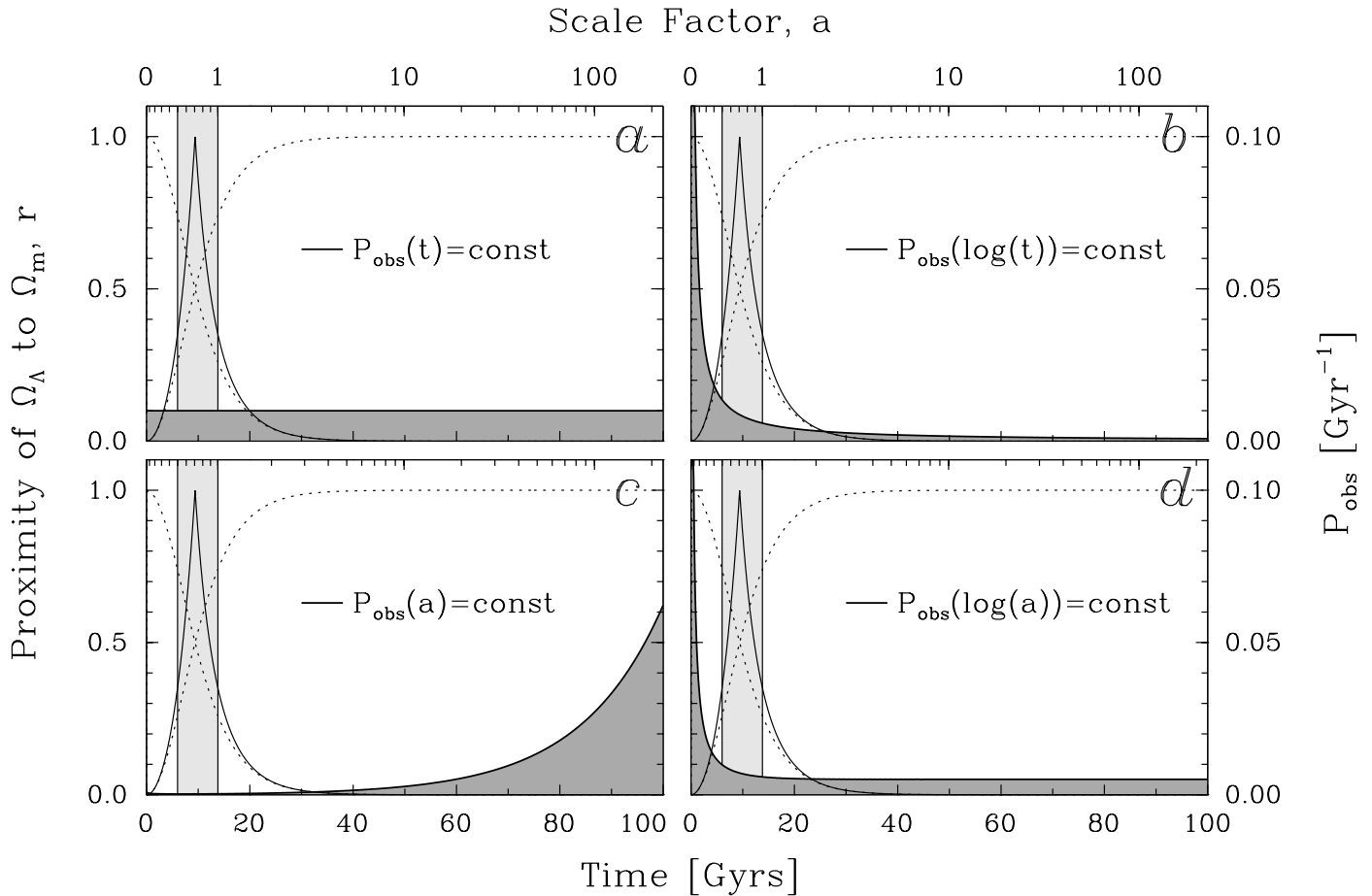
Fig. 8.—Demonstration of a variety of uniform observer distributions $P_{\rm obs}$ which, if used, result in the cosmic coincidence problem that the observed value of r is unexpectedly high. The expected observed value of r depends strongly on the assumed distribution of observers over time t. The $P_{\rm obs}$ that are functions of log a or log t have been normalized to the interval $t_{\rm rec}$ to 100 Gyr. Panel a is the same as the top panel of Fig. 3. The probabilities that an observer would fall within the vertical light gray band ($r > r_0$) in panels a, b, c, and d are 8%, 7%, 0.2%, and 6%, respectively, and are given in the first row of Table 2.

the observation of recent biogenesis unobservable (Lineweaver & Davis 2002, 2003). That is, no observer in the universe can wake up to observerhood and find that their planet is only a few hours old. Thus, the timescale for the evolution of observers $\Delta t_{\rm obs} > 0$.

Our $68\%_{-10\%}^{+14\%}$ result was calculated under the assumption that evolution from a new terrestrial planet to an observer takes $\Delta t_{\rm obs} \sim 4$ Gyr. To determine how robust our result is to variations in $\Delta t_{\rm obs}$, we perform the analysis of § 2 for 0 Gyr $< \Delta t_{\rm obs} <$ 12 Gyr. The results are shown in Figure 7. Our $68\%_{-10\%}^{+14\%}$ result is the data point plotted at $\Delta t_{\rm obs} = 4$ Gyr. If life takes ~ 0 Gyr to evolve to observerhood, once a terrestrial planet is in place $P_{\rm obs}(t) \approx$ PFR(t), and 55% of new cosmologists would observe an r value larger than the $r_0 \approx 0.4$ that we actually observe today. If observers typically take twice as long as we did to evolve ($\Delta t_{\rm obs} \sim 8$ Gyr), there is still a large chance ($\sim$30%) of observing $r > r_0$. If $\Delta t_{\rm obs} > 11$ Gyr, $P_{\rm obs}(t)$ in Figure 5 peaks substantially after $r(t)$ peaks, and the percentage of cosmologists who see $r > r_0$ is close to zero (eq. [5]). Thus, if the characteristic time it takes for life to emerge and evolve into cosmologists is $\Delta t_{\rm obs} \lesssim 10$ Gyr, our analysis provides a plausible solution to the cosmic coincidence problem.

The Sun is more massive than 94% of all stars. Therefore, 94% of stars live longer than the $t_\odot \approx 10$ Gyr main-sequence lifetime of the Sun. This is mildly anomalous, and it is plausible that the Sun's mass has been anthropically selected. For example, perhaps stars as massive as the Sun are needed to provide the UV photons

to jump start and energize the molecular evolution that leads to life. If so, then ~ 10 Gyr is a rough upper limit to the amount of time a terrestrial planet with simple life has to produce observers. Even if the characteristic time for life to evolve into observers is much longer than 10 Gyr, as concluded by Carter (1983), this UV requirement that life-hosting stars have main-sequence lifetimes $\lesssim 10$ Gyr would lead to the extinction of most extraterrestrial life before it can evolve into observers. This would lead to observers waking to observerhood to find the age of their planet to be a large fraction of the main-sequence lifetime of their star; the time they took to evolve would satisfy $\Delta t_{\rm obs} \lesssim 10$ Gyr, and they would observe that $r \sim 1$ and that other observers are very rare. Such is our situation.

If we assume that we are typical observers (Vilenkin 1995a, 1995b, 1996a, 1996b) and that the coincidence problem must be resolved by an observer selection effect (Bostrom 2002), then we can conclude that the typical time it takes observers to evolve on terrestrial planets is less than 10 Gyr ($\Delta t_{\rm obs} < 10$ Gyr).

3.2. *Dependence on the Age Distribution of Terrestrial Planets*

The $P_{\rm obs}(t)$ used here (Fig. 5) is based on the SFR computed in Lineweaver (2001). There is general agreement that the SFR has been declining since redshifts $z \sim 2$. Current debate centers around whether that decline has only been since $z \sim 2$ or whether the SFR has been declining from a much higher redshift (Lanzetta et al. 2002; Hopkins & Beacom 2006; Nagamine et al. 2006; Thompson et al. 2006). Since Lineweaver (2001) assumed a

TABLE 2
Probability $P(r > r_0)$ of Observing $r > r_0$ Assuming a Uniform Distribution
of Observers P_{obs} in Linear Time, log(time), Scale Factor,
and log(scale factor) within the Range Δx Listed

Range Δx [a]		$P(r > r_0)$ (%)			
x_{min}	x_{max}	t	$\log t$	a	$\log a$
t_{rec}	100 Gyr[b]	8[c]	7	0.2	6
t_{Planck}	$t_{last\ stars}$	8×10^{-4}	0.6[d]	10^{-10^4}	10^{-3}
t_{Planck}	t_0	60[c]	0.6	50	1
t_{Planck}	t_{Planck}^{-1}	30	0.3	30	0.5[d]
t_{Planck}	$t_{heat\ death}$	10^{-188}	0.1	$10^{-10^{189}}$	10^{-188}
t_{rec}	$t_{proton\ decay}$	10^{-26}	1	$10^{-10^{27}}$	10^{-26}
t_{rec}	$t_{black\ holes}$	10^{-88}	0.4	$10^{-10^{89}}$	10^{-88}
t_{rec}	$t_{heat\ death}$	10^{-188}	0.2	$10^{-10^{189}}$	10^{-188}
$t_{1ststars}$	$t_{last\ stars}$	8×10^{-4}	6	10^{-10^4}	10^{-3}
$t_{1ststars}$	$t_{proton\ decay}$	10^{-26}	1	$10^{-10^{27}}$	10^{-26}
$t_{1ststars}$	$t_{black\ holes}$	10^{-88}	0.4	$10^{-10^{89}}$	10^{-88}
$t_{1ststars}$	$t_{heat\ death}$	10^{-188}	0.2	$10^{-10^{189}}$	10^{-188}

[a] See Table 1 for the times corresponding to cols. (1) and (2).
[b] The four values of $P(r > r_0)$ in the top row correspond to Fig. 8.
[c] These values correspond to the two panels of Fig. 3.
[d] These values correspond to the two panels of Fig. 2.

relatively high value for the SFR at redshifts above 2, this led to a relatively high estimate of the metallicity of the universe at $z \sim 2$, which corresponds to a relatively short delay (~ 2 Gyr) between the big bang and the first terrestrial planets. For the purposes of this analysis, the early-SFR-dependent uncertainty in the ~ 2 Gyr delay is degenerate with, but much smaller than, the uncertainty of Δt_{obs}. Thus, the variations of Δt_{obs} discussed above subsume the SFR-dependent uncertainty in $P_{obs}(t)$.

3.3. Dependence on Measure

In Figures 2 and 3 we illustrated how the importance of the cosmic coincidence depends on the range over which one assumes that the observation of r could have occurred. This involved choosing the range Δx shown on the x-axis in Figures 2 and 3. We also showed how the apparent significance of the coincidence depended on how one expressed that range, i.e., logarithmic in Figure 2 and linear in Figure 3. The coincidence seems most compelling when Δx is the largest and the problem is presented on a logarithmic x-axis. This dependence is a specific example of a "measure" problem (Aguirre & Tegmark 2005; Aguirre et al. 2006).

The measure problem is illustrated in Figure 8, where we plot four different uniform distributions of observers on a linear time axis. In Figure 8a $P_{obs}(t) = $ constant. That is, we assume that observers could find themselves anywhere between $t_{rec} = 380{,}000$ yr and 100 Gyr after the big bang, with uniform probability (dark gray). In Figure 8b we make the different assumption that observers are distributed uniformly in $\log t$ over the same range in time. This means, for example, that the probability of finding yourself between 0.1 and 1 Gyr is the same as between 1 and 10 Gyr. We plot this as a function of linear time and find that the distribution of observers (dark gray) is highest toward earlier times.

To quantify and explore these dependencies further, in Table 2 we take the duration when $r > r_0$ (call this interval Δx_r) and divide it by various larger ranges Δx (a range of time or scale factor). Thus, when the probability $P(r > r_0) = \Delta x_r/\Delta x$ is $\ll 1$, there is a low probability that one would find oneself in the interval Δx_r and the cosmic coincidence is compelling. However, when $P(r > r_0) \sim 1$ the coincidence is not significant.

In Figures 8a, 8b, 8c, and 8d the probability of us observing $r \geq r_0$ (finding ourselves in the light gray area) is 8%, 7%, 0.2%, and 6%, respectively. These values are given in the first row of Table 2, along with analogous values when 11 other ranges for Δx are considered. Probabilities corresponding to the four panels of Figures 2 and 3 are shown in Table 2. Our conclusion is that this simple ratio method of measuring the significance of a coincidence yields results that can vary by many orders of magnitude depending on the range (Δx) and measure (e.g., linear or logarithmic) chosen. The use of the nonuniform $P_{obs}(t)$ shown in Figure 4 is not subject to these ambiguities in the choice of range and measure.

4. DISCUSSION AND SUMMARY

Anthropic arguments to resolve the coincidence problem include Garriga et al. (2000) and Bludman & Roos (2001). Both use a semianalytical formalism (Gunn & Gott 1972; Press & Schechter 1974; Martel et al. 1998) to compute the number density, as a function of ρ_Λ, of objects that collapse into large galaxies. This is then used as a measure of the number density of intelligent observers. Our work complements these semianalytic models by using observations of the SFR to constrain the possible times of observation. Our work also extends this previous work by including the effect of Δt_{obs}, the time it takes observers to evolve on terrestrial planets. This inclusion puts an important limit on the validity of anthropic solutions to the coincidence problem.

Garriga et al. (2000) is probably the work most similar to ours. They take ρ_Λ as a random variable in a multiverse model with a prior probability distribution. For a wide range of ρ_Λ (prescribed by a prior based on inflation theory) they find approximate equality between the time of galaxy formation t_G, the time when Λ starts to dominate the energy density of the universe t_Λ, and now t_0. That is, they find that, within 1 order of magnitude, $t_G \sim t_\Lambda \sim t_0$. Their analysis is more generic but approximate in that it addresses the coincidence for a variety of values of ρ_Λ to an order of magnitude precision. Our analysis is more specific and empirical in that we condition on our universe and use the Lineweaver (2001) SFR-based estimate of the age distribution of terrestrial planets to reach our main result (68%).

To compare our result to that of Garriga et al. (2000), we limit their analysis to the ρ_Λ observed in our universe ($\rho_\Lambda = 6.7 \times 10^{-30}$ g cm^{-3}) and differentiate their cumulative number of galaxies which have assembled up to a given time (their eq. [9]). We find a broad time-dependent distribution for galaxy formation which is the analog of our more empirical and narrower (by a factor of 2 or 3) $P_{obs}(t)$.

We have made the most specific anthropic explanation of the cosmic coincidence using the age distribution of terrestrial planets in our universe and found this explanation fairly robust to the largely uncertain time it takes observers to evolve. Our main result is an understanding of the cosmic coincidence as a temporal selection effect if observers emerge preferentially on terrestrial planets in a characteristic time $\Delta t_{obs} < 10$ Gyr. Under these plausible conditions, we, and any observers in the universe who have evolved on terrestrial planets, should not be surprised to find $\Omega_{m_0} \sim \Omega_{\Lambda_0}$.

APPENDIX

EVOLUTION OF DENSITIES

In the standard ΛCDM model the density parameters ($\Omega_i \equiv \rho_i/\rho_{\mathrm{crit}}$) of radiation, matter, and vacuum energy are currently observed to be $\Omega_{r_0} \approx (4.9 \pm 0.5) \times 10^{-5}$, $\Omega_{m_0} \approx 0.26 \pm 0.03$, and $\Omega_{\Lambda_0} \approx 0.74 \pm 0.03$, respectively, and Hubble's constant is $H_0 = 71 \pm 3$ km s^{-1} Mpc^{-1} (Spergel et al. 2007; Seljak et al. 2006). The energy densities in relativistic particles ("radiation"; i.e., photons, neutrinos, and hot dark matter), nonrelativistic particles ("matter"; i.e., baryons and cold dark matter), and vacuum energy scale differently (Peacock 1999),

$$\rho_i \propto a^{-3(w_i+1)}, \tag{A1}$$

where $w_{\mathrm{radiation}} = 1/3$, $w_{\mathrm{matter}} = 0$, and $w_\Lambda = -1$ (Linder 1997). In a flat universe these add up to the critical density so the density parameters obey the constraint (Peacock 1999)

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1. \tag{A2}$$

Figure 1 (*top*) illustrates these different dependencies on scale factor and time in terms of densities, while Figure 1 (*bottom*) shows the corresponding normalized density parameters. A false vacuum energy $\rho_{\Lambda_{\mathrm{inf}}}$ is assumed between the Planck scale and the GUT scale. Our value for $\Omega_{\Lambda_{\mathrm{inf}}}$ is based on the constraint that at the GUT scale all the energy densities add up to $\rho_{\Lambda_{\mathrm{inf}}}$, which remains constant at earlier times.

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